6 Freezing Loads and Freezing Time Calculation

Gauri S. Mittal
University of Guelph, Guelph, Ontario, Canada

CONTENTS

I. INTRODUCTION

The purposes of food freezing are: (i) preservation of food; (ii) reducing the activity of enzymes and microorganisms; (iii) reducing the amount of liquid water for microbial growth; and (iv) reducing water activity (a_w) of foods. Many types of freezers are used for this purpose. Some of these are: (i) air blast freezers, batch, or continuous; (ii) still air freezers; (iii) belt freezers; (iv) spiral belt freezers; (v) fluidized bed freezers; (vi) plate freezers — a series of flat plates kept cool by circulating a coolant; (vii) liquid immersion freezers — chilled brine or glycol is used, can also be sprayed; and (viii) cryogenic freezers — liquid N_2 or liquid CO_2 is used (the boiling point for N_2 is $-196^\circ C$ and for CO_2 is $-79^\circ C$).

Product quality is influenced by ice-crystal size and configuration during the freezing operation. The advantages of fast freezing operation can be lost during the storage because of the formation of large ice crystals by joining small crystals. Hence, complete product freezing in the freezer is more important [1]. The process of ice-crystal formation is based on two operations: (1) nucleation or crystal formation — it influences the type of crystal structure formed in a food product and ice-crystal nucleation is created by supercooling below initial freezing point, similar to crystallization process; (2) rate of crystal growth — is also supercooling-driven, which depends on (i) diffusion rate of water molecules from the unfrozen solution to the crystal surface, (ii) the rate at which heat is removed, and (iii) temperature of the solution.

II. FREEZING LOAD

A. Calculation

Freezing load or enthalpy change (ΔH) to reduce the product temperature (T_i) from some level above the freezing point (T_F) to some desired final temperature (T) is given by

$$\begin{aligned}\Delta H = &\text{ sensible heat removed from the product solids } (\Delta H_s)\\ &+ \text{sensible heat removed from unfrozen water } (\Delta H_u)\\ &+ \text{enthalpy change due to latent heat } (\Delta H_L)\\ &+ \text{sensible heat removed from the frozen water } (\Delta H_I)\end{aligned}$$

$$\Delta H_s = M_s\, C_{ps}(T_i - T_F) + \int_{T_F}^{T} M_s C_{ps}\, dT \tag{6.1a}$$

$$\Delta H_u = M_u\, C_{pu}(T_i - T_F) + M_u\, C'_{pu}(T_F - T) \tag{6.1b}$$

$$\Delta H_L = M_I\, L_V \tag{6.1c}$$

$$\Delta H_I = M_I\, C_{pI}(T_F - T) \text{ or } \int_{T_F}^{T} M_I\, C_{pI}\, dT \tag{6.1d}$$

where M_s is the mass of solids, C_{ps} the specific heat of solids, M_u the mass of unfrozen water, C_{pu} the specific heat of unfrozen water, M_I the mass of ice or unfrozen water, L_v the latent heat of freezing, C_{pI} the specific heat of ice, and C'_{pu} the specific heat of unfrozen water below T_F. Enthalpy composition charts for different food materials using experimental data were provided [2,3]. One example is given in Figure 6.1.

B. Freezing Rate and Thermal Center

The absence of a consistent definition for the freezing time is one of the problems associated with the published literature on the freezing of foods. This problem arises mainly because foods do not freeze at a distinct temperature, but rather the phase change takes place over a range of temperatures. A definition of the freezing time requires a definition of the freezing point. A variable temperature distribution exists within the food product during the freezing process, giving different freezing times depending on the point within the product where the temperature is monitored. The "thermal center", defined as the location in the material which cools most slowly, is generally used as the reference location. The effective freezing time, defined by the International Institute of

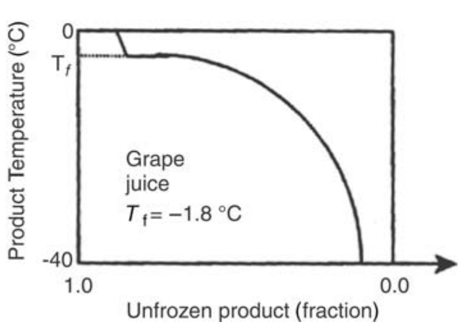

FIGURE 6.1 Riedel plot for grape juice. (From L Riedel, *Kaltetechnik* 9:38–40, 1957.)

Refrigeration [4], is the total time required to lower the temperature of a food material at its thermal center to a desired temperature below the initial freezing point. Other definitions are:

1. The time required to reduce the product temperature at the slowest cooling location from the initial freezing point to some desired and specified temperature below the initial freezing point.
2. International Institute of Refrigeration [4] definition: It is the ratio between the minimum distance from the surface to the thermal center and the time elapsed between the surface reaching 0°C and the thermal center reaching 5°C colder than the temperature of initial ice formation at the thermal center (cm/h).

III. FREEZING TIME OR RATE PREDICTION

It is important to accurately predict the freezing times of foods to assess the quality, processing requirements, and economical aspects of food freezing. A number of models have been proposed in the literature to predict freezing times. However, as the freezing process is a moving boundary problem, that is, one involving a phase change, most of the single-phase, unsteady-state solutions are unsuitable.

Foods, undergoing freezing, release latent heat over a range of temperatures. Freezing does not occur at a unique temperature. In addition, foods do not have constant thermal properties during freezing [5]. As a result, no exact mathematical model exists for predicting the freezing of foods. Researchers, who have found a solution, have either used numerical finite difference or finite element methods. So, models for predicting freezing times range from approximate analytical solutions to more complex numerical methods.

In the past, an extensive amount of work has been done to develop mathematical models for the prediction of food freezing times. The accuracy of such models is dependent on how closely the corresponding assumptions approach reality. Most of these models are usually categorized into one of two forms, analytical or numerical, with the latter generally considered as more accurate due to the inclusion of a set of assumptions and boundary conditions, which are of a more realistic nature than those pertaining to the former. Approximately, 30 different methods to predict freezing and thawing times were reviewed [6]. Details on these models are given elsewhere [7,8].

The general approach of researchers in the food-freezing field has been to seek approximate or empirical relationships, rather than to try to derive exact analytical equations. The method can be classified into two groups: (1) methods relying on analytical approximations, such as those of Refs. [9–13] or (2) methods relying on regression of computer results or experimental data, such as those of Refs. [14–18]. The methods vary considerably in complexity and accuracy, the number of arbitrary or empirical parameters used ranging from 0 to more than 50 [19].

A. Plank's Equation

Plank's equation was derived based on energy balance principle [9]. Heat condition through frozen region is written as: (Figure 6.2)

$$q = k_{\mathrm{I}} A \left(\frac{T_{\mathrm{S}} - T_{\mathrm{F}}}{X} \right) \tag{6.2}$$

Convective heat transfer at the surface is given by:

$$q = h_{\mathrm{c}} A \, (T_{\infty} - T_{\mathrm{S}}) \tag{6.3}$$

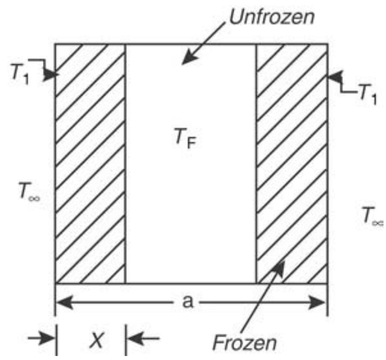

FIGURE 6.2 Diagram to derive Plank's equation.

Total resistance

$$R_t = \frac{X}{k_I A} + \frac{1}{h_c A} \tag{6.4}$$

or

$$q(\text{overall}) = \frac{\Delta T}{R_t} = \frac{T_\infty - T_F}{(X/k_I A + 1/h_c A)} \tag{6.5}$$

This heat transfer should be equal to the latent heat of freezing or

$$q = A \ \frac{dX}{dt} \rho L_V \tag{6.6}$$

$$\frac{dX}{dt} = \text{the velocity of the freezing front} \tag{6.7}$$

or

$$A \frac{dX}{dt} \rho L_V = -\frac{(T_\infty - T_F)A}{(X/k_I + 1/h_c)} (\text{negative heat transfer}) \tag{6.8}$$

or

$$\int_0^{t_F} dt = -\frac{L_V \rho}{T_\infty - T_F} \int_0^{a/2} \left(\frac{1}{h_c} + \frac{X}{k_I} \right) dX \tag{6.9}$$

or

$$t_F = -\frac{L_V \rho}{T_\infty - T_F} \left[\frac{a}{2h_c} + \frac{a^2}{8k_I} \right] = \frac{L_V \rho}{T_F - T_\infty} \left[\frac{a}{2h_c} + \frac{a^2}{8k_I} \right] \tag{6.10}$$

General form:

$$t_F = \frac{\rho L_V}{T_F - T_\infty} \left[\frac{Pa}{h_c} + \frac{Ra^2}{k_I} \right] \tag{6.11}$$

where T_F is the initial freezing point of the product, T_S the surface temperature, k_I the thermal conductivity of frozen food, X the thickness of frozen food, h_c the convective heat transfer coefficient, A the surface area, T_∞ the ambient temperature, L_v the latent heat of freezing, and ρ the food density.

P and *R* values for different shaped foods are:

	Infinite slab	Infinite cylinder	Sphere
P	1/2	1/4	1/6
R	1/8	1/16	1/24

For brick-shaped material, Figure 6.3 provides *P* and *R* for different β_1 and β_2 values [20].

Example 1: Lean beef block with dimensions of $1\,\text{m} \times 0.25\,\text{m} \times 0.6\,\text{m}$, $h_c = 30\,\text{W/(m}^2\,\text{K)}$, $T_0 = 5°\text{C}$, $T = -10°\text{C}$, $T_\infty = -30°\text{C}$, $\rho = 1050\,\text{kg/m}^3$, $L_V = 333.22\,\text{kJ/kg}$, m.c. = 74.5%, $k_I = 1.108\,(\text{W/m K})$, $T_F = -1.75°\text{C}$. Find freezing time using Plank's equation.

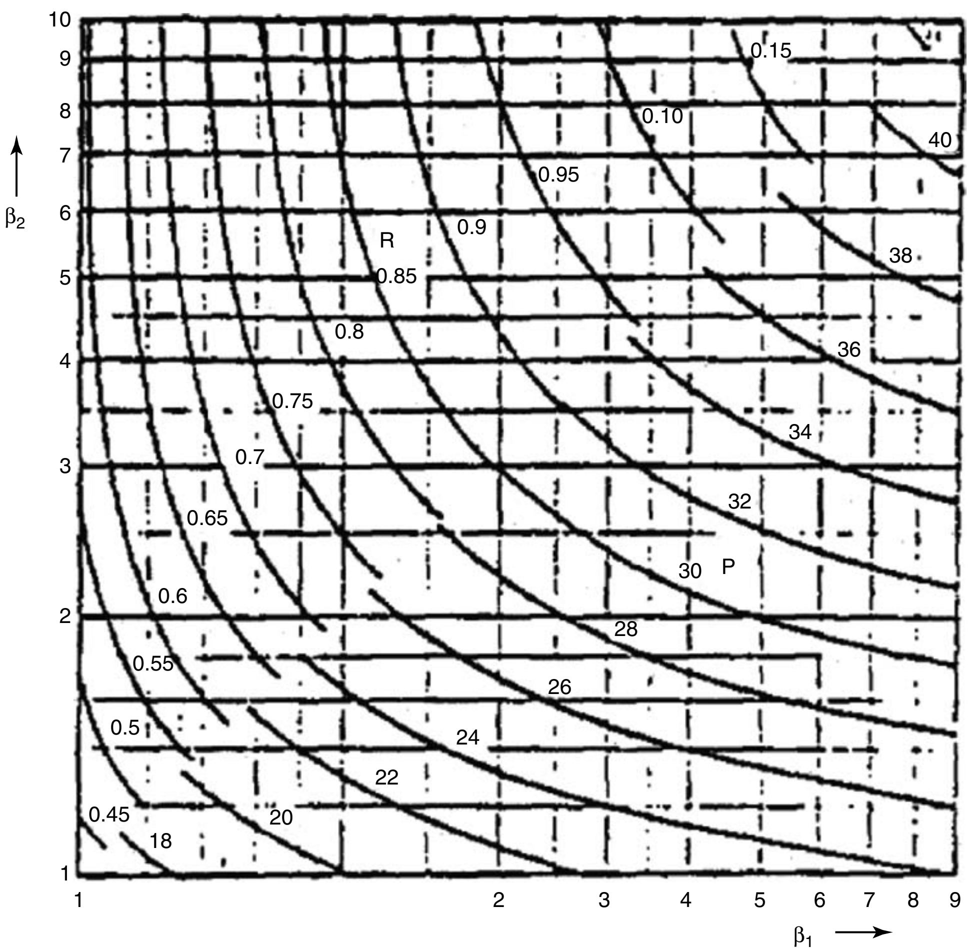

FIGURE 6.3 *P* and *R* values for different β_1 and β_2. (From AJ Ede. Modern Refrigeration 52:52-55, 1949.)

Solution:

$$\beta_1 = \frac{0.6}{0.25} = 2.4, \quad \beta_2 = \frac{1}{0.25} = 4, \quad \therefore P = 0.3, \quad R = 0.085$$

$$t = \frac{(1050)(333.22 \times 0.745)(1000\,\mathrm{J/kJ})}{[-1.75 - (-30)]3600\,\mathrm{s/h}}\left[\frac{0.3(0.25)}{30} + \frac{0.085(0.025)^2}{1.108}\right] = 18.7\,\mathrm{h}$$

The limitations of Plank's equation are as follows:

1. It neglects the time required to remove sensible heat above the initial freezing point.
2. It does not consider the gradual removal of latent heat over a range of temperatures during the freezing process.
3. Constant thermal conductivity assumed for frozen material.
4. It assumes the product to be completely in liquid phase.

Many modifications were suggested on Plank's equation to improve its accuracy. Some of these are given in the subsequent sections.

B. Nagaoka et al. Equation

Nagaoka et al. [21] proposed the modifications of Plank's equation as follows:

$$t_F = \frac{\Delta H'\rho}{T_F - T_\infty}\left[\frac{Pa}{h_c} + \frac{Pa^2}{k_I}\right] \tag{6.12}$$

$$\Delta H' = (1 + 0.008T_i)[C_{pu}(T_i - T_F) + L_V + C_{PI}(T_F - T)] \tag{6.13}$$

where T_i is the initial food temperature, T the final frozen food temperature, C_{pu} the specific heat of unfrozen food, and C_{PI} the specific heat of frozen food.

C. Levy Equation

Levy [22] considered the following definition of enthalpy to modify Plank's equation:

$$\Delta H' = (1 + 0.008(T_i - T_F))[C_{pu}(T_i - T_F) + L_V + C_{PI}(T_F - T)] \tag{6.14}$$

Example 2: Use modified Plank's equation to calculate the freezing time for the lean beef block of $1\,\mathrm{m} \times 0.6\,\mathrm{m} \times 0.25\,\mathrm{m}$, $h_c = 30\,\mathrm{W/(m^2K)}$, $T_0 = 5°\mathrm{C}$, $T = -10°\mathrm{C}$, $T_\infty = -30°\mathrm{C}$, $\rho = 1050\,\mathrm{kg/m^3}$, $T_F = -1.75°\mathrm{C}$, $t_F = ?$

$$\Delta H = 333.22\ \mathrm{kJ/kg}\ (0.745\,\mathrm{m.c.}) = 248.25\ \mathrm{kJ/kg},$$

Solution:

$$\beta_2 = \frac{0.6}{0.25} = 2.4, \quad \beta_1 = \frac{1}{0.25} = 4, \quad \therefore P = 0.3, \quad R = 0.085$$

$$C_{pu} = 3.52\,\mathrm{kJ/kg\,K}, \quad C_{PI} = 2.05\ \mathrm{kJ/kg\,K}, \quad k_I = 1.108\,\mathrm{W/m\,K}$$

$$\Delta H' = \{1 + 0.008[5 - (-1.75)]\}[3.52(5 - (-1.75)) + 248.25 + 2.05(-1.75 - (-10))]$$
$$= 297.59\,\text{kJ/kg}$$

$$t_f = \frac{(1050)(297.59)(1000)}{[-1.75 - (30)]3600}\left[\frac{0.3(0.25)}{30} + \frac{0.085(0.025)^2}{1.108}\right] = 22.41\,\text{h}$$

D. Cleland and Earle Equation

Cleland and Earle [23] modified Plank's equation using the nondimensional numbers as follows:

$$N_{\text{Ste}} = \text{Stefan number} = \frac{C_{\text{PI}}(T_F - T_\infty)}{\Delta H_{\text{ref}}} \tag{6.15}$$

$$t_F = \frac{\rho \Delta H_{\text{ref}}}{E(T_F - T_\infty)}\left[\frac{Pa}{h_c} + \frac{Ra^2}{k_I}\right]\left(1 - \frac{1.65 N_{\text{Ste}}}{k_I}\ \ln\left[\frac{T - T_\infty}{T_{\text{ref}} - T_\infty}\right]\right) \tag{6.16}$$

$$N_{\text{PK}} = \text{Plank's number} = \frac{C_{\text{pu}}(T_i - T_F)}{\Delta H_{\text{ref}}} \tag{6.17}$$

$$P = 0.5[1.026 + 0.5808 N_{\text{PK}} + N_{\text{Ste}}(0.2296 N_{\text{PK}} + 0.105)] \tag{6.18}$$

$$R = 0.125[1.202 + N_{\text{Ste}}(3.410 N_{\text{PK}} + 0.7336)] \tag{6.19}$$

where T_{ref} is the reference temperature and E is 1 for an infinite slab, 2 for an infinite cylinder, and 3 for a sphere. T_{ref} is taken as $-10°$C and ΔH_{ref} is enthalpy change from T_F to T_{ref}. $0.15 \le N_{\text{Ste}} \le 0.35, 0.2 \le N_{\text{Bi}} \le 20,$ and $0 \le N_{\text{PK}} \le 0.55$.

Example 3 (Cleland and Earle [23] approach): Lamb steak (slab) 0.025 m thick, $T_i = 20°$C, $T = -10°$C, $T_\infty = -30°$C, $\rho = 1050\,\text{kg/m}^3$, $T_F = -2.75°$C, $k_I = 1.35\,\text{W/m K}$, $h_c = 20\,\text{W/(m}^2\,\text{K)}$, $E = 1$ for slab, $t_F = ?$ $C_{\text{pu}} = 3\,\text{kJ/kg K}$, $C_{\text{PI}} = 1.75\,\text{kJ/kg K}$, $\Delta H = 240\,\text{kJ/kg}$.

Solution:

$$\Delta H_{\text{ref}} = 240 + 1.75(-2.75 + 10) = 252.7\,\text{kJ/kg}$$

$$N_{\text{Ste}} = \frac{C_{\text{PI}}(T_F - T_\infty)}{\Delta H_{\text{ref}}} = \frac{1.75(-2.75 + 30)}{252.7} = 0.189$$

$$N_{\text{PK}} = \frac{C_{\text{pu}}(T_i - T_F)}{\Delta H_{\text{ref}}} = \frac{3(20 + 2.75)}{252.7} = 0.270$$

$$P = 0.5[1.026 + 0.5808(0.270) + 0.189(0.2296(0.270) + 0.105)] = 0.607$$

$$R = 0.125[1.202 + 0.189(3.410(0.270) + 0.7336)] = 0.189$$

$$t_F = \frac{\rho \Delta H_{\text{ref}}}{T_F - T_\infty}\left[\frac{Pa}{h_c} + \frac{Ra^2}{k_I}\right]\left(1 - \frac{1.65\, N_{\text{Ste}}}{k_I}\ln\left[\frac{T - T_\infty}{T_{\text{ref}} - T_\infty}\right]\right)$$
$$= \frac{252.7(1000)(1050)}{(-2.75 + 30)3600}\left[\frac{0.607(0.025)}{20} + \frac{0.189(0.025)^2}{1.35}\right](1) = 2.289\,\text{h}$$

E. Cleland et al. Method

Cleland et al. [24,25] method is based on Calvelo [26] approach, which is given below:

$$t_f = \frac{1.3179 \rho C_{\text{PI}} a^2}{k_I E}\left[\frac{0.5}{N_{\text{Bi}} N_{\text{Ste}}} + \frac{0.125}{N_{\text{Ste}}}\right]^{0.9576} N_{\text{Ste}}^{0.0550}\, 10^{0.0017 N_{\text{Bi}} + 0.1727 N_{\text{PK}}}$$
$$\times \left[1 - \frac{1.65 N_{\text{Ste}}}{k_I}\ln\frac{T - T_\infty}{T_{\text{ref}} - T_\infty}\right] \tag{6.20}$$

T_{ref} is also $-10°\text{C}$. N_{Bi} is given by ha/k_I.

F. Pham Method

Pham method [13] involves total of precooling, phase change, and tempering times.

$$t_f = \frac{1}{E}\sum_{i=1}^{3} \Delta H_i a \frac{(1 + N_{\text{Bi}_i}/a_i)}{2\Delta T_i h_c} \tag{6.21}$$

where

$$\Delta H_1 = C_{\text{pu}}(T_i - T_{f,\text{ave}}) \tag{6.22}$$

$$\Delta T_1 = \frac{(T_i - T_\infty) - (T_{f,\text{ave}} - T_\infty)}{\ln((T_i - T_\infty)/(T_{f,\text{ave}} - T_\infty))}, \quad a_1 = 6 \tag{6.23}$$

$$N_{\text{Bi}_i} = 0.5\left(\frac{h_c a}{k_I} + \frac{h_c a}{k_u}\right) \tag{6.24}$$

$$\Delta H_3 = C_{\text{PI}}(T_{f,\text{ave}} - T_{\text{ave}}), \quad N_{\text{Bi}_3} = N_{\text{Bi}_2} \tag{6.25}$$

$$\Delta T_3 = \frac{(T_{f,\text{ave}} - T_\infty) - (T_{\text{ave}} - T_\infty)}{\ln((T_{f,\text{ave}} - T_\infty)/(T_{\text{ave}} - T_\infty))} \tag{6.26}$$

$$\Delta H_2 = \Delta H_f, \quad \Delta T_2 = T_{f,\text{ave}} - T_\infty, \quad N_{\text{Bi}_2} = \frac{h_c a}{k_I}, \quad a_2 = 4 \tag{6.27}$$

$$T_{\text{ave}} = T - \frac{T - T_\infty}{2 + 4/N_{\text{Bi}_3}}, \quad a_3 = 6, \quad T_{f,\text{ave}} = T_F - 1.5 \tag{6.28}$$

where k_W is the thermal conductivity of unfrozen food.

G. Modified Pham Method

This modified method of Pham [19] was given after summing precooling, phase change, and tempering times. E is given by the literature [27]. This method is to calculate the freezing and thawing time for finite size objects of any shape by approximating them to be similar to an ellipsoid. The following assumptions were used in developing this method: (i) uniform initial product temperature, T_i; (ii) uniform and constant ambient conditions; (iii) a fixed value of final product temperature, T; and (iv) convective surface heat transfer is following Newton's law of cooling.

For infinite slab, the freezing time (t_{slab}) is given by

$$t_{slab} = \frac{\rho a}{2h_c}\left[\frac{\Delta H_1}{\Delta T_1} + \frac{\Delta H_2}{\Delta T_2}\right]\left[1 + \frac{N_{Bi}}{4}\right] \tag{6.29}$$

Equation (6.21) is valid for the following ranges: $0.02 < N_{Bi} < 11, 0.11 < N_{Ste} < 0.36$, and $0.03 < N_{PK} < 0.61$.

The thawing time is given by for thawing to $T_f = 0°C$:

$$t_{slab} = \frac{1.4921 C_{pu} a^2}{k_u}\left[\frac{0.5}{N_{Bi}N_{Ste}} + \frac{0.125}{N_{Ste}}\right]^{1.0248} N_{Ste}^{0.2712} N_{PK}^{0.061} \tag{6.30}$$

Equation (6.30) is valid for the following ranges: $0.3 < N_{Bi} < 41$, $0.08 < N_{Ste} < 0.77$, and $0.06 < N_{PK} < 0.27$.

where

$$\Delta H_1 = C_{pu}(T_i - T_3) \tag{6.31}$$

$$\Delta H_2 = \Delta H + C_{PI}\,(T_3 - T) \tag{6.32}$$

$$\Delta T_1 = \frac{T_i - T_3}{2} - T_\infty, \quad \Delta T_2 = T_3 - T_\infty, \quad \Delta T_3 = 1.8 + 0.263T + 0.105T_\infty \tag{6.33}$$

$$N_{Bi} = h_c a / k_I \tag{6.34}$$

$$N_{Ste} = \frac{C_{PI}\,(T_F - T_\infty)}{\Delta H_{ref}} \tag{6.35}$$

$$N_{PK} = \frac{C_{pu}\,(T_i - T_F)}{\Delta H_{ref}} \tag{6.36}$$

where C_{pu} is the specific heat of unfrozen product (J/(kg K)), C_{PI} the specific heat of the frozen product (J/(kg K)), h_c the convective heat transfer coefficient (W/(m^2 K)), T_∞ the ambient temperature (°C), T_F the initial freezing temperature (°C), ρ the product density (kg/m^3), k_I the thermal conductivity of frozen product (W/(m K)), k_u the thermal conductivity of unfrozen product (W/(m K)), ΔH the enthalpy change due to freezing = (moisture content) (333 220) (J/kg), R the characteristic dimension (m), that is radius of cylinder of sphere or half thickness of slab or other geometries, N_{Bi} the Biot number, N_{Ste} the Stefan number, N_{PK} the Plank number, and T the final product temperature (°C).

For other shapes than infinite slab, the following modification is used:

$$t_{ellipsoid} = \frac{t_{slab}}{E} \tag{6.37}$$

$$E = 1 + \frac{1 + 2/N_{Bi}}{\beta_1^2 + 2\beta_1/N_{Bi}} + \frac{1 + 2/N_{Bi}}{\beta_2^2 + 2\beta_2/N_{Bi}} \tag{6.38}$$

where V is the volume (m^3) and A the smallest cross-sectional area that incorporate R (m^2).

For an infinite slab, E is 1, for an infinite cylinder E is 2, and for a sphere E is 3.

$$\beta_1 = \frac{A}{\pi R^2} \quad \text{and} \quad \beta_2 = \frac{V}{\beta_1(4/3\pi R^3)} \tag{6.39}$$

Notes:

1. For Equations (6.11), (6.12), (6.16), (6.20), (6.21), (6.29), (6.30), and (6.34), a is slab thickness or diameter of cylinder or sphere, or the smallest dimension of brick-shaped or dissimilar products.
2. $\Delta H =$ (moisture content) (latent heat of fusion); $\Delta H'$ for modified Plank's equations such as Levy's [22] and Nagaoka et al. [21], and $\Delta H_{10} = \Delta H + C_{\mathrm{PI}}(T_{\mathrm{F}} - 10)$.

Example 4: Beef slab of $1\,\mathrm{m} \times 0.6\,\mathrm{m} \times 0.25\,\mathrm{m}$, $R = 0.25/2 = 0.125\,\mathrm{m}$, $h_{\mathrm{c}} = 30\mathrm{W/(m^2\,K)}$, $\rho = 1050\,\mathrm{kg/m^3}$, $T_{\mathrm{i}} = 5°\mathrm{C}$, $T = -15°\mathrm{C}$, $T_\infty = -30°\mathrm{C}$, $C_{\mathrm{PI}} = 2.5\,\mathrm{kJ/(kg\,K)}$, $C_{\mathrm{pu}} = 3.52\,\mathrm{kJ/(kg\,K)}$, moisture content = 74.5% wet basis, $\Delta H = 333.22(0.745) = 248.25 = \mathrm{kJ/kg}$, m.c. = 74.5%, $k_{\mathrm{I}} = 1.108(\mathrm{W/mK})$, $T_{\mathrm{F}} = -1.75°\mathrm{C}$, $t = ?$

Solution:

$$N_{\mathrm{Bi}} = \frac{h_{\mathrm{c}}a}{k_{\mathrm{I}}} = \frac{30(0.125)}{1.108} = 3.3845$$

$$N_{\mathrm{Ste}} = \frac{C_{\mathrm{PI}}(T_{\mathrm{F}} - T_\infty)}{\Delta H_{\mathrm{ref}}} = \frac{2.05(-1.75 + 30)}{248.25} = 0.234$$

$$N_{\mathrm{PK}} = \frac{C_{\mathrm{pu}}(T_{\mathrm{i}} - T_{\mathrm{F}})}{\Delta H_{\mathrm{ref}}} = \frac{3.52(5 + 1.75)}{248.25} = 0.0955$$

$$T_3 = 1.8 + 0.263T + 0.105T_\infty = 1.8 + 0.263(-15) + 0.105(-30) = -5.295$$

$$\Delta H_1 = C_{\mathrm{pu}}(T_{\mathrm{i}} - T_3) = 3520(5 + 5.295) = 36238.4\,\mathrm{J/kg}$$

$$\Delta H_2 = \Delta H + C_{\mathrm{PI}}(T_3 - T) = 248250 + 3520(-5.295 + 15) = 282411.6\,\mathrm{J/kg}$$

$$\Delta T_1 = \frac{T_{\mathrm{i}} - T_3}{2} - T_\infty = \frac{(5 + 5.295)}{2} - (-30) = 35.1475$$

$$\Delta T_2 = T_3 - T_\infty = -5.295 + 30 = 24.705$$

The freezing time (t_{slab}) is given by

$$t_{\mathrm{slab}} = \frac{\rho R}{h_{\mathrm{c}}}\left[\frac{\Delta H_1}{\Delta T_1} + \frac{\Delta H_2}{\Delta T_2}\right]\left[1 + \frac{N_{\mathrm{Bi}}}{2}\right] = \frac{1050(0.125)}{30}\left[\frac{36238.4}{35.1475} + \frac{282411.6}{24.705}\right]\left(1 + \frac{3.3845}{2}\right)$$
$$= 146789.45\,\mathrm{s} = 40.775\,\mathrm{h}$$

$$\beta_1 = \frac{A}{\pi R^2} = \frac{0.25(0.6)}{\pi(0.125)^2} = 3.056$$

$$\beta_2 = \frac{V}{\beta_1\left(\frac{4}{3}\pi R^3\right)} = \frac{0.25(0.6)(1)}{\frac{4}{3}\pi(3.056)(0.125)^3} = 6.0$$

Actual freezing time is given by

$$t_{\text{ellipsoid}} = \frac{t_{\text{slab}}}{E}$$

$$E = 1 + \frac{1 + 2/N_{\text{Bi}}}{\beta_1^2 + 2\beta_1/N_{\text{Bi}}} + \frac{1 + 2/N_{\text{Bi}}}{\beta_2^2 + 2\beta_2/N_{\text{Bi}}}$$

$$= 1 + \frac{1 + 2/3.3845}{3.056^2 + 2(3.056)/3.3845} + \frac{1 + 2/3.3845}{6^2 + 2(6)/3.3845} = 1.4939$$

Therefore, $t = 40.775/1.4939 = 27.294$ h.

IV. THAWING TIME PREDICTION

Although thawing is the opposite process of freezing, the earlier equations on freezing time prediction cannot be readily applied to thawing process. The thawing time is given for thawing to $T_{\text{F}} = 0°\text{C}$, and can be calculated by one of the following methods.

1. Power law approach to modifying Plank's equation as proposed by Calvelo [26] and Cleland [28]: This and other methods are valid for the following ranges: $0.6 < N_{\text{Bi}} < 57.3$, $0.08 < N_{\text{Ste}} < 0.77$, and $0.06 < N_{\text{PK}} < 0.27$.

$$t_{\text{slab}} = \frac{1.4921\rho C_{\text{pu}} a^2}{k_{\text{u}}} \left[\frac{0.5}{N_{\text{Bi}} N_{\text{Ste}}} + \frac{0.125}{N_{\text{Ste}}}\right]^{1.0248} N_{\text{Ste}}^{0.2712} N_{\text{PK}}^{0.061} \tag{6.40}$$

where

$$N_{\text{Bi}} = \frac{h_{\text{c}} a}{k_{\text{I}}} \tag{6.41}$$

$$N_{\text{Ste}} = \frac{C_{\text{PI}}(T_\infty - T_{\text{F}})}{\Delta H_{10}} \tag{6.42}$$

$$N_{\text{PK}} = \frac{C_{\text{pu}}(T_{\text{F}} - T_{\text{i}})}{\Delta H_{10}} \tag{6.43}$$

Here ΔH_{10} is the enthalpy change for the temperature change from 0 to $-10°\text{C}$.

2. Linear correction [23]:

$$t = \frac{\rho C_{\text{pu}} a^2}{k_{\text{I}} E} \left[\frac{P}{N_{\text{Bi}} N_{\text{Ste}}} + \frac{R}{N_{\text{Ste}}}\right] \tag{6.44}$$

$$P = 0.5[0.7754 + 2.2828 N_{\text{Ste}} N_{\text{PK}}] \tag{6.45}$$

$$R = 0.125[0.4271 + 2.1220 N_{\text{Ste}} - 1.4847 N_{\text{Ste}}^2] \tag{6.46}$$

3. Three-stage calculation method [13]:

$$t = \frac{\rho}{E} \sum_{i=1}^{3} \Delta H_{\text{i}} a \frac{(1 + ha/4k_{\text{I}})}{2\Delta T_{\text{i}} h_{\text{c}}} \tag{6.47}$$

where

$$\Delta H_1 = C_{\text{pu}}(T_{\text{f,ave}} - T) \tag{6.48}$$

$$\Delta T_1 = T_\infty - \frac{(T_{\text{i}} + T_{\text{f,ave}})}{2}, \quad k_1 = k_{\text{I}} \tag{6.49}$$

$$\Delta H_3 = C_{\text{pu}}(T_{\text{ave}} - T_{\text{f,ave}}) \tag{6.50}$$

$$\Delta T_3 = T_\infty - \frac{(T_{\text{ave}} + T_{\text{f,ave}})}{2}, \quad k_3 = k_{\text{u}} \tag{6.51}$$

$$\Delta H_2 = \Delta H_{\text{f}} \tag{6.52}$$

$$\Delta T_2 = T_\infty - T_{\text{f,ave}}, \quad k_2 = 0.25k_{\text{I}} + 0.75k_{\text{u}} \tag{6.53}$$

$$T_{\text{f,ave}} = T_{\text{F}} - 1.5 \tag{6.54}$$

$$\Delta T_{\text{ave}} = T - \frac{(T - T_\infty)}{2 + 4/N_{\text{Bi}}}, \tag{6.55}$$

4. Correction of Plank's equation [13]:

$$t_{\text{t}} = \frac{\rho C_{\text{pu}} a^2}{k_{\text{u}} E}\left[\frac{1}{2N_{\text{Bi}}N_{\text{Ste}}} + \frac{1}{8N_{\text{Ste}}}\right]\left[0.8941 - \frac{0.0244}{N_{\text{Ste}}} + \frac{0.6192N_{\text{PK}}}{N_{\text{Bi}}}\right] \times \left[1 + \frac{C_{\text{pu}}\,(T_{\text{ave}} - T)}{\Delta H_{10}}\right] \tag{6.56}$$

IV. CONCLUSIONS

Many equations and models have been suggested to calculate freezing time of foods. Whenever a freezing time prediction method is used, some inaccuracy will be inevitable. This may arise from one of the three sources: (a) inaccuracy in thermal data; (b) inaccurate knowledge of freezing conditions, particularly the surface heat transfer coefficient; and (c) inaccuracy arising from assumptions made in the derivation of the prediction equation. The best freezing time prediction method will be the one in which the error arising from the category (c) is the least. The method should require as few input data as possible, and preferably should avoid lengthy or complex operations or reference to grasp and table. Three important parameters affecting the freezing time prediction are L_{v}, h_{c}, and D. The parameter h_{c} is the most difficult one to measure accurately, and therefore, is a major source of error.

NOMENCLATURE

A	smallest cross-sectional area that incorporate R (m^2)
A	surface area (m^2)
C_{PI}	specific heat of the frozen product (J/(kg K))
C_{ps}	specific heat of solids (J/(kgK))
C'_{pu}	specific heat of unfrozen water below T_{F} (J/(kg K))
C_{pu}	specific heat of unfrozen product (J/(kg K))

E	shape factor
h_c	convective heat transfer coefficient (W/(m^2 K))
ΔH	enthalpy change due to freezing = (moisture content) (333220 J/kg)
ΔH	freezing load or enthalpy change (J/kg)
ΔH_I	sensible heat removed from the frozen water (J/kg)
ΔH_L	enthalpy change due to latent heat (J/kg)
ΔH_s	enthalpy change of product solids (J/kg)
ΔH_u	sensible heat removed from unfrozen water (J/kg)
ΔH_{10}	enthalpy change for the temperature change from 0 to -10°C J/kg)
k_I	thermal conductivity of frozen product (W/(m, K))
k_u	thermal conductivity of unfrozen product (W/(m, K))
k_W	thermal conductivity of unfrozen food (W/(m, K))
L_v	latent heat of freezing (J/kg)
M_I	mass of ice or unfrozen water (kg)
M_S	mass of solids (kg)
M_u	mass of unfrozen water (kg)
N_{Bi}	Biot number
N_{PK}	Plank number
N_{Ste}	Stefan number
R	characteristic dimension (m), that is radius of cylinder of sphere or half thickness, of slab or other geometries
t_{slab}	freezing time (s)
T	final frozen product temperature (°C)
T_F	initial freezing temperature (°C)
T_i	initial food temperature (°C)
T_{ref}	reference temperature (°C)
T_S	surface temperature (°C)
T_∞	ambient temperature (°C)
V	volume (m^3)
X	thickness of frozen food (m)
ρ	product density (kg/m^3)

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